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# Vertical Distribution of Dilute Suspensions in Turbulent Pipe Flow

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**Closed-form expressions are presented for predicting vertical concentration distribution of the dispersed phase in the turbulent core of pipe and channel flow. Experimental data obtained with spherical particles, of mean diameter $d_{50}$ comparable to the Kolmogorov microscale, are in good agreement with model predictions. The average value of dimensionless lateral particle diffusivity, $\zeta = \epsilon/Ru_*$, determined from these data is approximately 0.25, that is, fairly close to previously reported measurements with much coarser particles.**

## SCOPE

In steady horizontal pipe flow, the density difference between a dispersed solid or liquid phase and the continuous fluid phase can cause a nonuniform distribution of the dispersion in the pipe cross section. From a theoretical point of view, an accurate method for calculating particle or droplet concentration distribution can be very helpful in modeling dispersed two-phase flow systems. From a practical standpoint, prediction of flow conditions associated with large concentration gradients is often desirable, for example, in the design and operation of transfer lines in process units and of long distance pipelines transporting slurries. The particular problem motivating the present work is the development of flow criteria for a sufficiently uniform distribution of sediment and water in crude oil pipelines which are necessary for capturing a representative sample.

A simple diffusion type of model, proposed many years ago (O'Brien, 1933; Rouse, 1937) for vertical sediment distribution in open channels and rivers is usually applied to steady flow in closed conduits as well. This model, however, has some serious limitations, especially in the case of relatively concentrated suspensions of a wide size spectrum. Hunt (1954, 1969) has presented a more complete description of solids distribution in turbulent flow and has applied it to open channel flow. The main objective of this study was to use Hunt's formulation for pipe flow and to develop a general expression, predicting concentra-

tion profiles, which could be applied to suspensions of wide particle size distribution and possibly of relatively high concentration. Experimental data were also obtained, with dilute suspensions of spherical particles, in order to test the validity of the new model and to determine the lateral particle diffusivity in pipe flow.

## CONCLUSIONS AND SIGNIFICANCE

Closed-form expressions [Equations (27) and (32)] have been obtained for predicting the vertical concentration distribution of the dispersed phase in the turbulent core of pipe and channel flow. This new theory is based on a diffusion type of formulation, with a dimensionless lateral diffusivity, $\zeta = \epsilon/Ru_*$, independent of space coordinates. Accurate data obtained with spherical particles dispersed in hydrocarbons of kinematic viscosities 1.5 to 17 mm$^2$/s in pipes with diameters 5.04 and 7.53 cm are in very good agreement with the model predictions. Particle diffusivities determined from these data show no significant effect of particle diameter, friction velocity, or fluid viscosity on $\zeta$. The arithmetic average diffusivity obtained from twenty-three profiles is $\zeta = 0.255$, which is three to four times larger than the diffusivity of fluid in the absence of particles.

In most of our experiments, the median particle diameter $d_{50}$ was nearly equal to the Kolmogorov microscale of turbulence $\eta$. Other researchers have used spherical particles of diameter one to two orders of magnitude larger than $\eta$. Yet, they have obtained only slightly higher diffusivities, that is, $\zeta \simeq 0.3$ to 0.4. It appears, therefore, that the lateral particle diffusivity is rather weakly dependent on particle diameter $d$ for $d \geqq \eta$.

The theoretical expressions obtained in this study are rather general in that they predict the composite concentration profile as well as the distribution of each size fraction in the pipe cross section. Predicted and measured profiles show that considerable particle segregation can take place under steady flow conditions at sufficiently small velocities; that is, the mean particle size is much smaller at the top of the conduit than at the bottom. Under these conditions, the composite concentration profiles are asymmetric, with the local concentration at the center line smaller than the average $\overline{C}$.

Equations (27) and (32) are expected to be useful in engineering applications as well as in research studies of dilute dispersed flow systems. Ongoing research in our laboratory also shows that these expressions provide a very satisfactory framework for predicting gravitational effects in very concentrated slurries.

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Over the last 50 yr a great deal of work has been done on the prediction of particle concentration distribution in turbulent streams, mainly by civil engineers concerned with sediment transport in rivers and open channels. The approximate diffusion equation (O'Brien, 1933; Rouse, 1937)

$$\epsilon \frac{dC}{dy} + wC = 0 \qquad (1)$$

has found wide application for such calculations. Equation (1) simply states that the flux of solid particles due to gravity $wC$ is balanced by the upward flux due to turbulent diffusion $\epsilon\ (dC)/(dy)$.

This formulation has been criticized (see, for example, Snyder and Lumley, 1971) as fundamentally incorrect for diffusion processes characterized by a length scale comparable to the size of the diffusing species. In some cases, especially those involving large particles, this criticism is certainly valid. Nevertheless, it has already been demonstrated that the relatively simple gradient diffusion formulation provides a reliable and convenient tool for studying many aspects of particle dispersion in turbulent flow. For instance, Batchelor (1966), in a stimulating review of particle motion in turbulent streams, suggests that Equation (1) can provide criteria for determining whether particles stay in suspension almost all the time or reside in the constant stress layer at the bottom of an open channel. Elder (1959) has also used the same formulation in his study of longitudinal particle dispersion in pipe flow.

Aside from the above basic objections, the completeness of the model represented by Equation (1) has been questioned by Hunt (1969). He has shown that the main deficiency of this model lies in the fact that the law of mass conservation has been essentially disregarded by not taking into account the volume of particles in suspension. Hunt has developed a more complete set of diffusion equations, in which he has taken into account the liquid volume displaced upwards by an equal volume of particles moving in the lower strata of a horizontal flow field. Obviously, this formulation is more suitable than Equation (1) for studies of relatively concentrated suspensions. Another major advantage of Hunt's equations is that they can be directly applied to suspensions of a wide particle size distribution, as will be discussed in more detail later. So far this new formulation has been applied only to open channel flow (Hunt, 1954). In this work, it is used to obtain a closed-form expression for the vertical distribution of small, heavy particles in horizontal pipe flow.

In most of the previous experimental studies on particle distribution in closed conduits, Equation (1) has been employed to determine the transverse diffusivity $\epsilon$; for example, Ismail (1952), Binnie and Phillips (1958), Sharp and O'Neill (1971). A few investigators, notably Soo and his students, have proposed more elaborate formulations, assuming a certain form of $\epsilon$. For instance, Soo and Tung (1971) have used particle momentum equations in the axial direction in addition to a diffusion type of equation.

At present there is considerable uncertainty regarding the magnitude of the transverse particle diffusivity $\zeta$ in the core of pipe flow. Binnie and Phillips (1958) and Barnard and Binnie (1963) obtained data on particle mean velocity and lateral diffusivity by measuring the velocity of individual particles and their location in the pipe cross

section. Barnard and Binnie's measurements showed that $\zeta \simeq 0.30$. Binnie and Phillips found satisfactory correlation of their data with $\zeta \simeq 0.46$. Recently, Sharp and O'Neill (1971) obtained data with dilute suspensions of 2.54 mm particles. Their measurements showed that over most of the pipe cross section, characterized by 0.8R, the particle concentration is a function of the vertical coordinate $y$ only, and that for almost neutrally buoyant particles $\zeta \simeq 0.38$. As Batchelor (1966) has pointed out, it is quite surprising that the value of $\zeta$ is much larger than ~0.07, which is obtained on the basis of the Reynolds analogy. Furthermore, it is questionable whether this large discrepancy is a genuine manifestation of the complex particle-fluid interactions or an erroneous result due to the inappropriate model employed to estimate $\zeta$, for the previously outlined reasons.

All the above experiments were performed with relatively large particles (2.5 to 5.0 mm in diameter) in water. In this study, a considerable amount of data was obtained with spherical particles approximately ten times smaller and three fluids with kinematic viscosity 1.5 to 17.0 mm²/s. With such small particles one can better justify the use of a diffusion type of model to predict concentration distribution. Therefore, the dual purpose of these experiments was to test the validity of the new theoretical expression for predicting concentration profiles and to obtain an accurate estimate of particle diffusivity.

## THEORY

### General Formulation

For convenience, we shall outline the development of equations (Hunt, 1969) describing vertical diffusion of particles. The flux vector of particles of a given size $d_j$ and volume fraction $c_j$ is given by

$$\mathbf{p}_j = \mathbf{u}_j c_j - \epsilon_j \nabla c_j, \quad j = 1, 2, \ldots n \tag{2}$$

Equation (2) implies that the classical gradient diffusion law applies to turbulent diffusion of solid particles. The flux for the liquid flow is

$$\mathbf{q} = \mathbf{v}\left(1 - \sum_i c_i\right) - \epsilon_l \nabla\left(1 - \sum_i c_i\right) \tag{3}$$

where $\sum_i c_i$ is the composite local concentration. The relationship between liquid and particle velocity vectors is given by

$$\mathbf{u}_j = \mathbf{v} - w_j \mathbf{y} \tag{4}$$

where y is in a direction opposite to that of the gravitational field. For steady conditions, the following equations of continuity are obtained:

$$\nabla \cdot \mathbf{p}_j = 0, \quad j = 1, 2, \ldots n \tag{5}$$

$$\nabla \cdot \mathbf{q} = 0 \tag{6}$$

If the particle distribution is a function only of $y$ (horizontal flow) and there is no particle deposition (zero particle flux in the $y$ direction), then Equations (4), (5), and (6) reduce to

$$(v_y - w_j)c_j - \epsilon_j \frac{\partial c_j}{\partial y} = 0, \quad j = 1, 2, \ldots n \tag{7}$$

$$v_y\left(1 - \sum_i c_i\right) + \epsilon_l \frac{\partial}{\partial y} \sum_i c_i = 0 \tag{8}$$

Rearranging the above system of equations and assuming that the particle diffusivities $\epsilon_i$ are uniform, for example equal to $\epsilon$, we get

$$\epsilon \frac{\partial c_j}{\partial y} + c_j\left[w_j - \frac{\epsilon_l \sum_i w_i c_i}{\epsilon + (\epsilon_l - \epsilon)\sum_i c_i}\right] = 0, \quad j = 1, 2, \ldots n \tag{9}$$

$$v_y = \frac{\epsilon_l \sum_i w_i c_i}{\epsilon + (\epsilon_l - \epsilon)\sum_i c_i} \tag{10}$$

This system can be further simplified if $\epsilon_l \simeq \epsilon$; that is

$$\epsilon \frac{\partial c_j}{\partial y} + c_j\left[w_j - \sum_{i=1}^{n} w_i c_i\right] = 0, \quad j = 1, 2, \ldots n \tag{11}$$

$$v_y = \sum_{i=1}^{n} w_i c_i \tag{12}$$

As was discussed earlier, experimental results show that the lateral particle diffusivity $\epsilon$ is greater than that of Newtonian fluids ($\epsilon_l$). Nevertheless, the above approximation is still valid in view of the fact that the factor $\epsilon_l\left[\epsilon + (\epsilon_l - \epsilon)\sum_{i=1}^{n} c_i\right]$ in the last term of Equation (9) is of order 1, and, for sufficiently small concentrations, $w_j >> \sum_{i=1}^{n} w_i c_i$.

The system of Equations (11) will be used here to develop closed-form expressions for the particle distribution $c_j(y)$. The total or composite concentration at a point $y$ is given by

$$C(y) = \sum_{i=1}^{n} c_i(y) \tag{13}$$

For suspensions of uniform particle size, the system (11) reduces to

$$\epsilon \frac{dC}{dy} + (1 - C)Cw = 0 \tag{14}$$

It will be noticed that the Rouse Equation (1) is an approximate form of Equation (14), and it is valid at very small concentrations at which $C(1 - C) \simeq C$.

**Pipe Flow.** For horizontal conduits, the system of Equations (11) admits the following general solution:

$$c_j(y) = \frac{G_j \cdot \exp[-w_j f(y)]}{1 + \sum_i G_i \cdot \exp[-w_i f(y)]}; \quad j = 1, 2, \ldots n \tag{15}$$

where $f(y) = \int dy/\epsilon(y)$ and $G_j$ is a set of coefficients characteristic of each size fraction but independent of the space coordinates. In order to proceed in the development of the distribution function $c_j(y)$, the following two assumptions will be made: the dimensionless eddy diffusivity

$\zeta$ is a constant independent of flow conditions and space coordinates; that is

$$\epsilon = \zeta R u_*, \tag{16}$$

and the solids concentration is a function of the vertical coordinate $y$ only. Experiments by Sharp and O'Neill (1971) in a 5.08 cm I.D. pipe show that the above two assumptions are quite realistic, except very close to the wall. Using Equation (16) for $\epsilon$, we obtain

$$f(y) = \int_0^y \frac{dy}{\zeta u_* R} = \frac{y'}{\zeta u_*} \tag{17}$$

where $y' = y/R$, $+1 \geq y' \geq -1$. Substitution of Equation (17) into (15) leads to

$$c_j(y') = \frac{G_j \exp(-K_j y')}{1 + \sum_i G_i \exp(-K_i y')}; \quad j = 1, 2, \ldots n \tag{18}$$

where

$$K_j \equiv \frac{w_j}{\zeta u_*}, \quad j = 1, 2, \ldots n \tag{19}$$

A transformation is required in order to determine the parameters $G_j$. First, we define the following local and mean relative concentrations

$$v_j(y) = \frac{c_j(y)}{1 - \sum_i c_i(y)}, \quad \overline{v_j} = \frac{\overline{c_j}}{1 - \sum_i \overline{c_i}} = \frac{\overline{c_j}}{1 - \overline{C}} \tag{20}$$

where the bars denote quantities averaged over the pipe cross section. In terms of the relative concentrations $v_j$, the solution (18) can be expressed as follows:

$$v_j(y') = G_j \exp(-K_j y'); \quad j = 1, 2, \ldots n \tag{21}$$

Now it is convenient to apply the condition that, at steady state, the mean concentration of each particle size $\overline{c_j}$ in the pipe cross section is constant and known a priori. Of course, the relative concentration $\overline{v_j}$ is constant, too. Therefore, integration of Equation (21) over the pipe cross-sectional area A leads to

$$\overline{v_j} \equiv \frac{1}{A}\int_A v_j(y')dA = G_j \frac{1}{A}\int_A \exp(-K_j y')dA$$

or

$$G_j = \frac{\overline{v_j}}{E(K_j)}; \quad j = 1, 2, \ldots n \tag{22}$$

with the coefficients $E(K_j)$ defined as follows:

$$E(K_j) \equiv \frac{1}{A}\int_A \exp(-K_j y')dA; \quad j = 1, 2, \ldots n \tag{23}$$

This integral can be written in terms of the angular coordinate $\theta = \cos^{-1}\left(\frac{y}{R}\right)$ as follows:

$$E(K_j) = \frac{2}{\pi}\int_0^{\pi} [\exp(-K_j \cos\theta)][\sin^2\theta]d\theta \tag{24}$$

Furthermore, it can be shown that

$$E(K_j) = 2\frac{I_1(K_j)}{K_j} \tag{25}$$

where $I_1(K_j)$ is the well-known modified Bessel function of order 1, given in tabulated form by Abramowitz and Stegun (1965). Another approximate form of $E(K_j)$ can be obtained by expanding $\exp(-K_j y')$ in a convergent power series of $K_j y'$, for small values of $K_j y'$, which permits evaluation of the integral (23) in terms of elementary functions. The result is

$$E(K_j) = 1 + \frac{K_j^2}{8}\left[1 + \frac{K_j^2}{24}\right] + O(K_j^6) \tag{26}$$

Despite the fact that the above expansion is valid for $K_j y' < 1$, Equation (26) is in error by only $-0.5\%$ at $K_j y' = 2.0$ and $-3.5\%$ at $K_j y' = 3.0$.

The final solution for the concentration distribution is

$$c_j(y) = \left[\frac{\overline{v_j}}{E(K_j)}\exp(-K_j y')\right] \cdot \left[1 + \sum_{i=1}^{n} \frac{\overline{v_i}}{E(K_i)}\exp(-K_i y')\right]^{-1}; \quad j = 1, 2, \ldots n \tag{27}$$

where $E(K_j)$, $K_j$, and $\overline{v_j}$ are given by Equations (25) or (26), (19), and (20), respectively. The mean volumetric concentration of each size fraction $\overline{c_j}$ as well as the mean composite concentration $\overline{C}$ are considered known a priori. It is obvious from the above solution that the parameter $K_j$ and especially the ratio $w_j/u_*$ of the settling velocity to the friction velocity essentially determines the magnitude of the vertical concentration gradients. The significance of this ratio has been recognized and discussed in previous studies. If the particle size is uniform, or nearly uniform, then the solids distribution is given by the following simplified form of Equation (27):

$$c(y) = \left[1 + \frac{E(K)}{\overline{V}}\exp(Ky')\right]^{-1} \tag{28}$$

Here $\overline{V} = \overline{C}/(1 - \overline{C})$. It can be easily verified that this expression (28) is the solution to the simple diffusion Equation (14).

**One-Dimensional Channel Flow.** The vertical concentration distribution in turbulent flow through a closed channel of rectangular cross section, in addition to its practical significance, is of interest because of the one-dimensionality of the flow field if the channel is sufficiently wide. In this case, the assumption of concentration variation only in the vertical direction is made without any reservations, and equations for $c_j$, presented below, are expected to be much more accurate than those for pipe flow.

We consider a channel of height $h$ and eddy diffusivity given by

$$\epsilon = \xi h u_* \tag{29}$$

The general solution (15) also applies in this case with

$$f(y) = \int_0^y \frac{dy}{\xi h u_*} = \frac{y'}{\xi u_*}; \quad y' = \frac{y}{h} \tag{30}$$

The vertical coordinate varies in this case between 0 and 1 and the parameter $K_j'$ is defined as follows

$$K_j' \equiv \frac{w_j}{\xi u_*}; \quad j = 1, 2, \ldots n \tag{31}$$

Following the same procedures, outlined above for pipe flow, we obtain the final solution

$$c_j(y') = \left[\frac{\overline{v_j}}{J(K_j')}\exp(-K_j' y')\right] \cdot$$

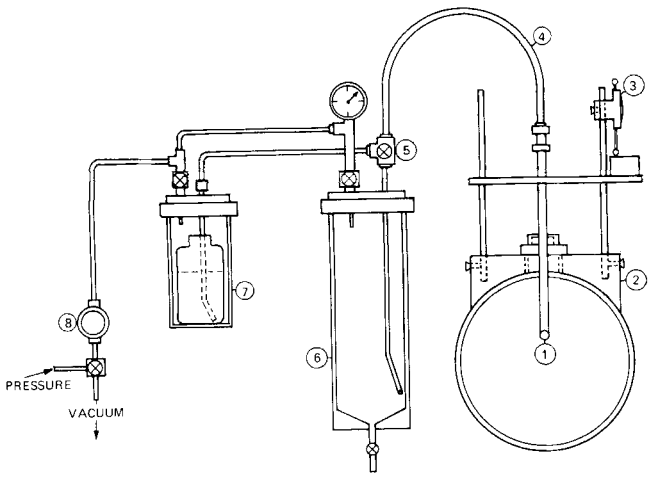


Fig. 1. Schematic of sampling system.

$$\left[ 1 + \sum_{i=1}^{n} \frac{\overline{v_i}}{J(K_i')} \exp\left(-K_i' y'\right) \right]^{-1}; \quad j = 1, 2, \ldots n \qquad (32)$$

where

$$J(K_j') \equiv [1 - \exp(-K_j')]; \quad j = 1, 2, \ldots n \qquad (33)$$

In the case of uniform size particles, the above solution reduces to Equation (28), with $J(K')$ instead of $E(K)$.

## DESCRIPTION OF EXPERIMENTS

Tests were carried out in a flow loop of total length approximately 22 m. Most of the pipe loop was 3.17 cm I.D., in which particle distribution was practically uniform in the range of flow rates of our experiments. Two test sections, made of smooth acrylic plastic, were used with mean I.D. 7.53 and 5.04 cm. Sections of different diameter were connected with special swages in order to minimize flow disturbances. Sampling units were located 65 to 70 pipe diam downstream of such connections. The rotary type of pump used (manufacturer Bowie Co., 500 series) did not cause appreciable particle attrition. Similarly, a 0.12 m$^3$ stainless steel mixing vessel was carefully designed to achieve uniform particle distribution with no significant particle breakage. A turbine type of meter (manufacturer Halliburton Co.) was employed for flow rate measurements.

A special sampling unit (Figure 1) was fabricated in order to obtain representative samples from the pipeline under nearly isokinetic conditions. An L shaped, 6.35 mm O.D., thin wall, stainless steel tube (1) was used to withdraw the sample. A saddle (2) was glued on the test section to support the sampling tube and the associated depth adjusting device. The probe's vertical position was measured with a precision Starrett dial indicator (3). Flexible nylon tubing (4) connected the probe to the rest of the sampling unit.

The sample was collected in a 300 cm$^3$ plastic bottle, placed in a Plexiglass vessel (7). A graduated cylinder (6) was used for measuring the withdrawal flow rate under constant backup pressure in order to determine the mean velocity through the sampling tube. The flow could be directed either to the collection vessel (7) or to the auxiliary cylinder (6) by means of a three-way ball valve (5). In order to obtain a representative sample, the velocity of withdrawal was adjusted to be nearly equal to, and slightly higher than, the local velocity in the pipe. The required constant backup pressure was controlled by a Grove (model 15L) regulator (8). At low pipeline velocities, it was sometimes necessary to provide vacuum, instead of pressure, to control the sampling flow rate.

Adjustment of the sampling rate was fairly time consuming. Therefore, only five to seven samples, at various vertical locations in each section, could be collected under constant conditions. The fluid volume collected in each bottle was approximately 250 cm$^3$ and contained particles on the order of 1 g, sufficient for accurate sample analysis. Particle concentration in these samples was determined gravimetrically. Particle size analyses were made with standard sieves and acetone as the washing fluid.

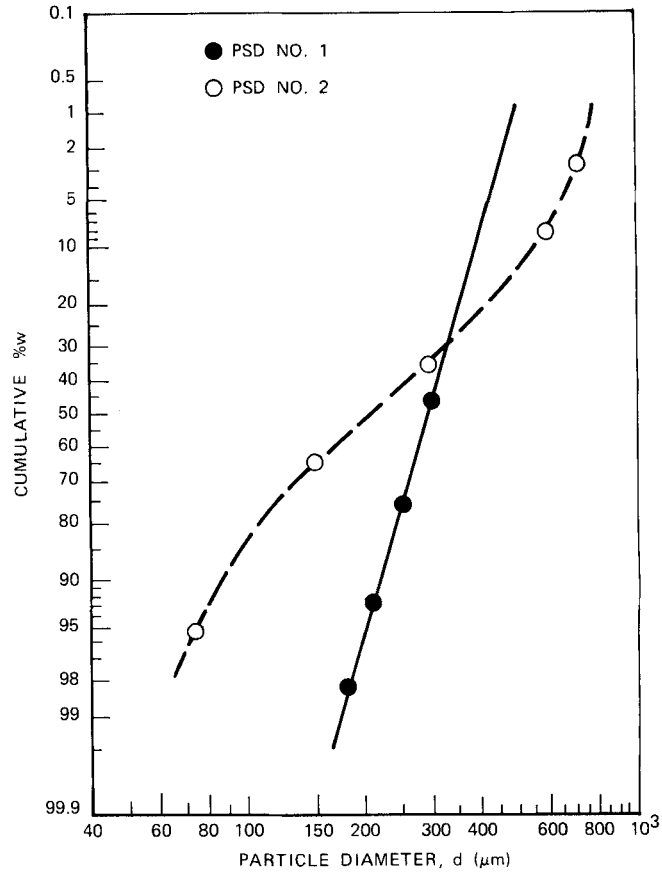


Fig. 2. Particle size distributions.

Small plastic particles were used in the tests. They were analytical grade anion exchange resin beads (manufacturer Bio Rad Laboratories, Richmond, California) of spherical shape and very smooth surface. Figure 2 shows the two different particle size distributions (PSD) employed in our tests. PSD No. 1 is a narrow log-normal distribution with $d_{50} \simeq 290$ μm; PSD No. 2 is rather wide with $d_{50} \simeq 210$ μm and somewhat more representative of droplet or particle size spectra encountered in physical processes. Owing to the finite elasticity and the spherical shape of the beads, as well as other precautions taken in the flow system, no particle breakage took place during the tests. Therefore, the size distributions of Figure 2 remained essentially unchanged throughout the experiments. In order to eliminate particle agglomeration and swelling due to absorbed moisture, the beads were dried before they were mixed with the test fluids. The density of dry beads measured in a Le Chatelier flask with kerosene was $1.126 \pm 0.004$ g cm$^{-3}$.

Three different fluids were used: kerosene, transformer oil (commercial name Diala), and a mixture of 33% vol kerosene-67% Diala which will be referred to as kerosene-Diala. In order to prevent the development of static charges on the acrylic pipe sections, a minute quantity of an effective antistatic compound was added to all fluid batches.

Experimental conditions are summarized in Tables 1 and 2. The dispersed phase mean concentration was kept constant at ~0.3% in all tests. Data were taken only along the vertical pipe diameter. Despite the very low level of mean particle concentration, the results were very consistent and reproducible. For example, data from run Nos. 4 and 7, taken under nearly identical conditions, showed excellent reproducibility. Also, the average concentration $\overline{C}$ obtained by integrating the measured profiles was in almost all cases within 10% of the initial overall average concentration.

## COMPARISON OF DATA WITH THEORY

The theoretical expression obtained in this paper fits all the measured concentration profiles vary well, with a di-

Table 1. Summary of Experimental Conditions: Experiments with Particle Size Distribution No. 1

| Run No. | Pipe I.D., cm | Mean velocity, m/s | Mean temp., °C | Fluid properties $\rho$, g/cm³ | Fluid properties $\mu$, mN s/m² | Average solids fraction by volume, $\bar{C} \times 10^3$ | Remarks |
|---|---|---|---|---|---|---|---|
| 3 | 7.53 | 0.945 | 24.2 | 0.807 | 1.79 | 3.072 | Kerosene used in all runs |
| 4 | 7.53 | 1.219 | 23.9 | 0.807 | 1.80 | 3.182 | |
| 5 | 7.53 | 0.655 | 24.4 | 0.806 | 1.78 | 3.054 | |
| 6 | 7.53 | 1.433 | 27.5 | 0.804 | 1.68 | 3.159 | |
| 7 | 7.53 | 1.219 | 22.8 | 0.808 | 1.84 | 3.185 | |
| 8 | 5.04 | 1.494 | 21.9 | 0.808 | 1.87 | 3.374 | |
| 9 | 5.04 | 2.134 | 23.1 | 0.807 | 1.83 | 3.224 | |
| 10 | 7.53 | 1.722 | 27.5 | 0.804 | 1.68 | 3.331 | |

Table 2. Summary of Experimental Conditions: Experiments with Particle Size Distribution No. 2

| Run No. | Pipe I.D., cm | Mean velocity, m/s | Mean temp., °C | Fluid properties $\rho$, g/cm³ | Fluid properties $\mu$, mN s/m² | Average solids fraction by volume, $\bar{C} \times 10^3$ | Remarks |
|---|---|---|---|---|---|---|---|
| 11 | 7.53 | 0.686 | 21.9 | 0.808 | 1.86 | 2.852 | Kerosene |
| 12 | 7.53 | 1.067 | 23.3 | 0.807 | 1.82 | 3.132 | Kerosene |
| 13 | 7.53 | 1.372 | 25.8 | 0.805 | 1.73 | 3.135 | Kerosene |
| 14 | 7.53 | 1.570 | 26.4 | 0.805 | 1.72 | 3.324 | Kerosene |
| 15 | 5.04 | 1.524 | 20.8 | 0.809 | 1.90 | 3.221 | Kerosene |
| 16 | 5.04 | 1.829 | 20.8 | 0.809 | 1.90 | 3.141 | Kerosene |
| 17 | 5.04 | 2.134 | 22.5 | 0.808 | 1.85 | 3.065 | Kerosene |
| 18 | 5.04 | 2.438 | 23.6 | 0.807 | 1.81 | 3.159 | Kerosene |
| 19 | 7.53 | 0.686 | 21.9 | 0.893 | 15.70 | 3.275 | Diala |
| 20 | 7.53 | 1.082 | 23.9 | 0.892 | 14.40 | 3.323 | Diala |
| 21 | 7.53 | 1.372 | 24.2 | 0.892 | 14.25 | 3.457 | Diala |
| 22 | 5.04 | 1.524 | 24.4 | 0.892 | 14.10 | 3.371 | Diala |
| 23 | 7.53 | 0.686 | 20.3 | 0.865 | 6.45 | 3.174 | Kerosene-Diala |
| 24 | 7.53 | 1.082 | 21.4 | 0.864 | 6.25 | 3.278 | Kerosene-Diala |
| 25 | 7.53 | 1.372 | 23.9 | 0.863 | 5.57 | 3.240 | Kerosene-Diala |
| 26 | 5.04 | 1.402 | 21.7 | 0.864 | 6.17 | 3.234 | Kerosene-Diala |

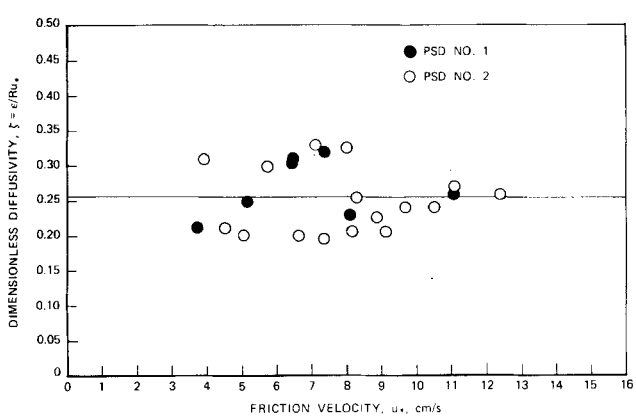


Fig. 3. Dimensionless particle diffusivities obtained in this study.

mensionless diffusivity $\zeta = \epsilon/Ru_*$ varying between 0.20 and 0.33. In Figure 3 the value of $\zeta$, which gives the best fit of each measured profile with Equation (27), is plotted vs. the corresponding friction velocity $u_*$. In general, these data show no systematic effect of $u_*$, particle size, or liquid viscosity on the diffusivity. The arithmetic average of the data presented in Figure 3 is $\zeta \simeq 0.255$, which is surprisingly close to the $\zeta$ values obtained by Barnard and Binnie (1963) and Sharp and O'Neill (1971) with much coarser particles. Furthermore, the values of particle diffusivity obtained in our study are three to five times greater than the diffusivity of momentum in the turbulent core.

Figures 4 and 5 show the effect of mean velocity on the vertical distribution of the dispersed phase for the two dif-

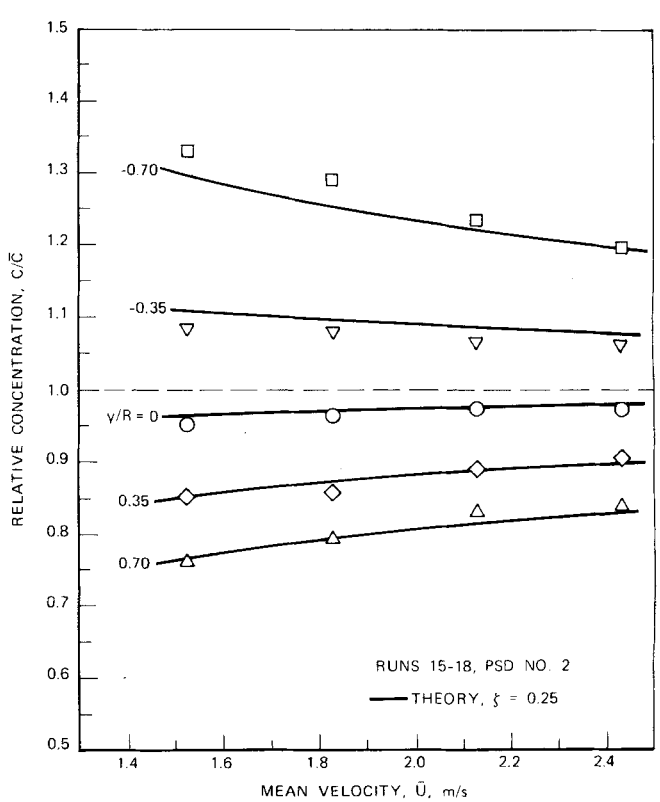


Fig. 4. Effect of mean velocity on vertical concentration distribution. $D = 5.04$ cm.

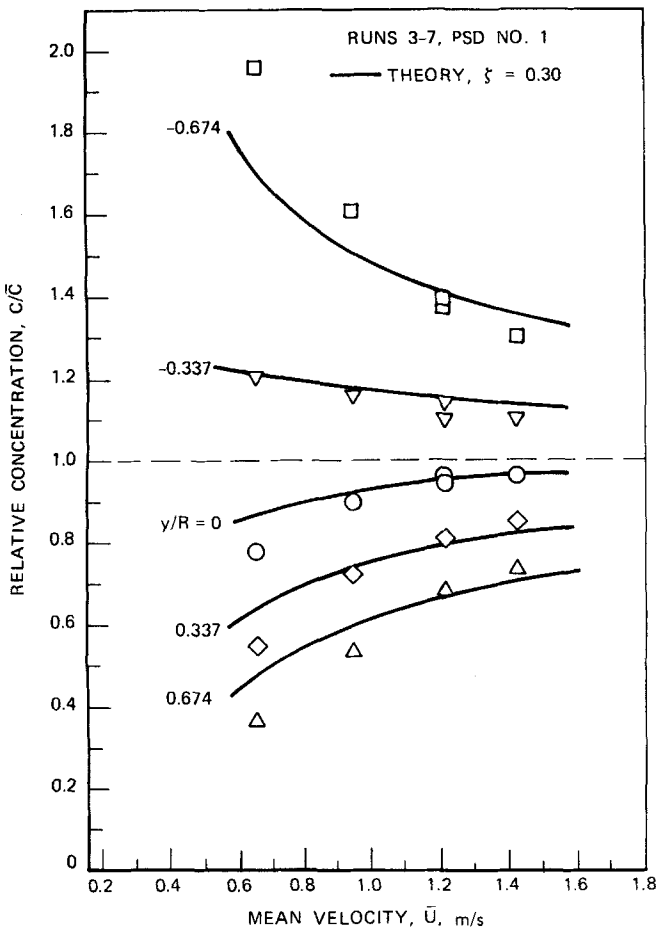


Fig. 5. Effect of mean velocity on vertical concentration distribution. $D = 7.53$ cm.

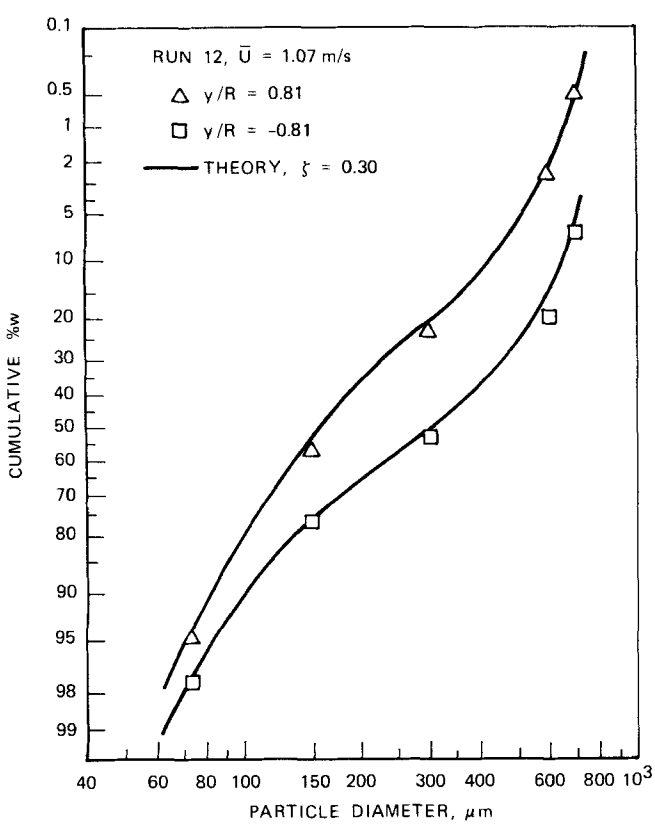


Fig. 7. Effect of gravity on particle segregation. D = 7.53 cm, particle size distribution No. 2.

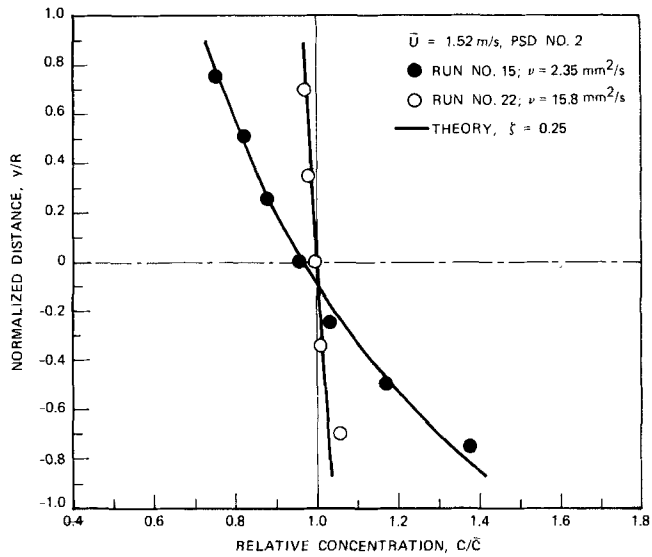


Fig. 6. Effect of viscosity on vertical concentration distribution. $D = 5.04$ cm.

ferent pipe diameters and particle size distributions used in our tests. There is very good agreement between data and predictions, with $\zeta = 0.25$ to 0.30. Of particular interest here is the large deviation, from the mean value $\overline{C}$, of the concentration close to the top and bottom of the pipe. Also, the concentration at the center line tends to decrease with velocity, and it is below the average even at fairly high velocities, as shown in Figure 4. The influence of viscosity on the concentration profile is clearly shown in Figure 6. At sufficiently high viscosities (for example, $\nu = 15$ mm$^2$/s), the dispersed phase is almost uniformly distributed, even at relatively small velocities.

Of special interest in the design of multiphase flow systems, and in modeling dispersed droplet or particle flows, is not only the time-average concentration profile but also the vertical variation of the size spectrum. Figure 7 shows that the model developed here can accurately predict particle segregation in a horizontal pipe caused by gravity. The size spectra plotted in Figure 7 show that the volume median diameter $d_{50}$ at $y/R = +0.81$, that is close to the pipe top, is approximately 150 $\mu$m, whereas at $y/R = -0.81$ $d_{50} \simeq 300$ $\mu$m.

Very few concentration profiles, at low solids fractions, have been reported in the literature. Of particular interest are the very asymmetric profiles obtained by Durand and Condolios (1952) in a 15 cm I.D. pipe with fine sand ($d \simeq 0.180$ mm) in water. In this often quoted study, a conductivity type of probe was employed to measure the local solids concentration. Figure 8 shows some data obtained at an estimated mean concentration $\overline{C} \simeq 0.065$.

According to Durand and Condolios, only the upper part of these profiles, that is, above the center line, can be predicted by the Rouse Equation (1). Lack of information about the size distribution and settling velocity of the particles used in that study does not permit a detailed analysis and comparison of data with our model predictions. However, on the assumption that the particles were of nearly uniform size, Equation (28) with a constant parameter $K$ is in very good agreement with the measured profiles, as shown in Figure 8. Furthermore, for an assumed mean settling velocity $w = 2.0$ to 2.5 cm/s, the dimensionless diffusivity $\zeta$ corresponding to these profiles is estimated to be between 0.08 and 0.12. These values are much smaller than the diffusivities obtained in this and other studies with spherical particles, but still 1.2 to 1.8 times the momentum diffusivity in the turbulent core.

Data obtained by Ismail (1952) in a closed channel of rectangular cross section can be used for comparison with Equation (32). On the basis of measured velocity profiles, Ismail determined that the momentum diffusivity over most of the channel cross section can be approximated as follows:

$$\epsilon_l = 0.11\, khu_* \tag{34}$$

For $k = 0.38$, the dimensionless momentum diffusivity is

$$\xi_l \equiv \frac{\epsilon_l}{hu_*} = 0.042 \tag{35}$$

Ismail's data also show that the ratio of particle to fluid diffusivity, $\beta = \xi/\xi_l$, is larger than unity.

Figure 9 shows data, obtained with a narrow size distribution of mean diameter ~100 $\mu$m, at the largest and smallest reported mean concentration $\overline{C}$. The values of $\overline{C}$ given in Figure 9 were computed by integrating the measured concentration profiles. There is excellent agreement between data and predictions based on Equation (32). In the theoretical calculations, the following average values were used, as reported by Ismail (Table 1, page 418):

Run 73 $\dfrac{w}{\beta k u_*} = 0.59$, $w = 0.814$ cm/s, $u_* = 2.865$ cm/s

Run 78 $\dfrac{w}{\beta k u_*} = 0.29$, $w = 0.884$ cm/s, $u_* = 6.645$ cm/s

For constant $k = 0.38$, these values correspond to dimensionless particle diffusivities $\xi = 0.053$ and 0.051, respectively, that is, 1.27 to 1.21 times the momentum diffusivity.

## DISCUSSION

The dimensionless particle diffusivity obtained in this study ($\zeta \simeq 0.25$) is several times larger than the fluid diffusivity in the absence of particles. Similar results were obtained in previous studies (Barnard and Binnie, 1963; Binnie and Phillips, 1958; Sharp and O'Neill, 1971) with much larger particles. In most of our experiments, the median particle diameter $d_{50}$ was of the same order of magnitude as the Kolmogorov microscale $\eta = (\nu^3/\bar{\epsilon})^{1/4}$, based on the mean rate of energy dissipation per unit mass $\bar{\epsilon} = 2f\overline{U}^3/D$. In the previous studies, the particle diameter was one to two orders of magnitude larger than the microscale $\eta$. It appears, therefore, that the diffusivity of spherical particles in pipe flow may not be strongly dependent on particle diameter $d$ if $d \geqslant \eta$.

Obviously, further studies are required to quantify the effect of particle properties on their diffusivity $\zeta$ and to provide a satisfactory explanation as to why $\zeta$ is so large compared to the diffusivity of the fluid. Studies of individual particle motions by Snyder and Lumley (1971) and Jones and his students (see, for example, Meek and Jones, 1973, and Howard, 1974) are very useful in this respect. They show that particle autocorrelation functions and associated Lagrangian integral scales can be very different from the corresponding fluid properties. Of particular significance to this study are the recent measurements by Howard (1974) in vertical pipe flow, showing that lateral particle dispersion ($\overline{x^2}$) generally increases with particle diameter $d$ for $d$ larger than the turbulence microscale. Also measurements by Kada and Hanratty (1960) show that for sufficiently large particle settling velocities and concentrations, the fluid diffusivity is substantially greater than that in the absence of solids.

From the viewpoint of engineering calculations, the theoretical expressions obtained in this study are rather

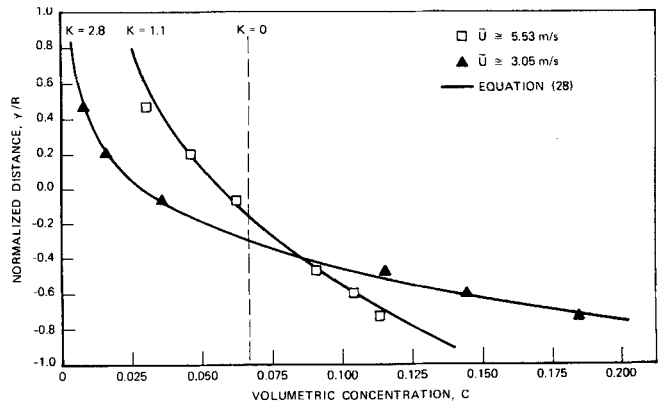


**Fig. 8. Vertical distribution of sand in water. Data obtained by Durand and Condolios (1952) in a 15 cm I.D. pipe; $\overline{C} \simeq 0.065$.**

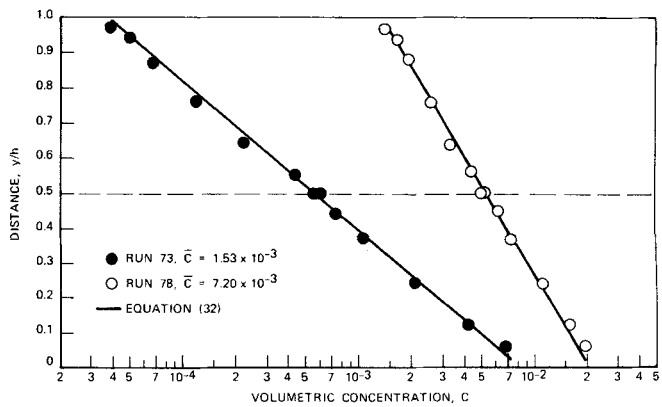


**Fig. 9. Comparison between theoretical predictions and data obtained by Ismail (1952) in a closed rectangular channel.**

general in that they predict the composite concentration profile $C(y')$ as well as the distribution of each size fraction in the pipe cross section. They can be also applied to suspensions of buoyant particles and to inclined pipes, at small angles of inclination, if the component of settling or rising velocity on the pipe cross section is used. However, from the previously outlined derivation, it is clear that they may have some limitations. The most significant limitation may result from the assumption of a constant particle diffusivity, which is not expected to be valid close to the wall. Nevertheless, the good agreement between data and predictions suggests that the new model is accurate at least in the turbulent core.

Experiments presently underway show that the model developed here can provide the framework for very satisfactory predictions of particle vertical profiles in the case of very concentrated slurries. Such predictions, in addition to their practical usefulness, are considered an essential step towards modeling flow of concentrated suspensions. These results will be reported in a future publication.


## ACKNOWLEDGMENT

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## NOTATION

$C$ = solids volumetric concentration, dimensionless
$\overline{C}$ = concentration averaged over pipe cross section, dimensionless
$c_i, \overline{c_i}$ = local and average concentration of particle size fraction $i$, dimensionless
$D$ = pipe inside diameter, L

$d$ = particle diameter, L
$d_{50}$ = median particle diameter; 50% wt of particles are larger than $d_{50}$, L
$f$ = friction factor, dimensionless
$h$ = channel height, L
$k$ = von Karman constant
$K_j$ = parameter; $w_j/\zeta u_*$, dimensionless
$K_j'$ = parameter; $w_j/\xi u_*$, dimensionless
$R$ = pipe inside radius, L
$\overline{U}$ = mean flow velocity, $LT^{-1}$
$u_*$ = friction velocity, $\overline{U}(f/2)^{1/2}$, $LT^{-1}$
$\mathbf{u}_j$ = particle velocity vector
$v_i$ = relative particle concentration; defined by Equation (20), dimensionless
$v_y$ = liquid velocity in the vertical direction $y$, $LT^{-1}$
$\mathbf{v}$ = liquid velocity vector
$w_i$ = particle settling velocity corresponding to $d_i$, $LT^{-1}$
$y'$ = reduced vertical coordinate, $y/R$, dimensionless
$\mathbf{y}$ = unit vector

**Greek Letters**

$\epsilon$ = diffusivity, $L^2T^{-1}$
$\epsilon_j$ = particle diffusivity, $L_2T^{-1}$
$\epsilon_l$ = diffusivity of liquid
$\bar{\epsilon}$ = mean rate of energy dissipation per unit mass of fluid; $2f\overline{U}^3/D$, $L^2T^{-3}$
$\zeta$ = dimensionless diffusivity in pipe flow; $\epsilon/Ru_*$, dimensionless
$\eta$ = Kolmogorov microscale; $(\nu^3/\bar{\epsilon})^{1/4}$, L
$\nu$ = fluid kinematic viscosity, $L^2T^{-1}$
$\xi$ = dimensionless diffusivity in closed channel flow; $\epsilon/hu_*$, dimensionless

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# Electrochemical Extraction of Copper From Scrap Steel


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**A packed-bed electrochemical method is presented for the removal of copper from scrap steel. The method is based on the principle that at certain potentials, steel can be passivated in a suitable electrolyte, whereas copper dissolves anodically. A rotating hemispherical electrode was used to determine the polarization curves of copper and mild steel in various electrolytes. Extraction tests were carried out with an electrochemical cell composed of a cathodic carbon packed bed and an anodic bed packed with synthesized steel scrap.**


## SCOPE

Disposal of ferrous solid waste is a national problem. Although total annual steel produced and steel scrapped have reached a steady state in the United States, steel scrap represents only 50% of the raw material for the production of iron and steel. The amount of obsolete scrap not recycled was over 37 million tons in 1970 (Albrecht and McDermott, 1973). One of the major technical obstacles


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